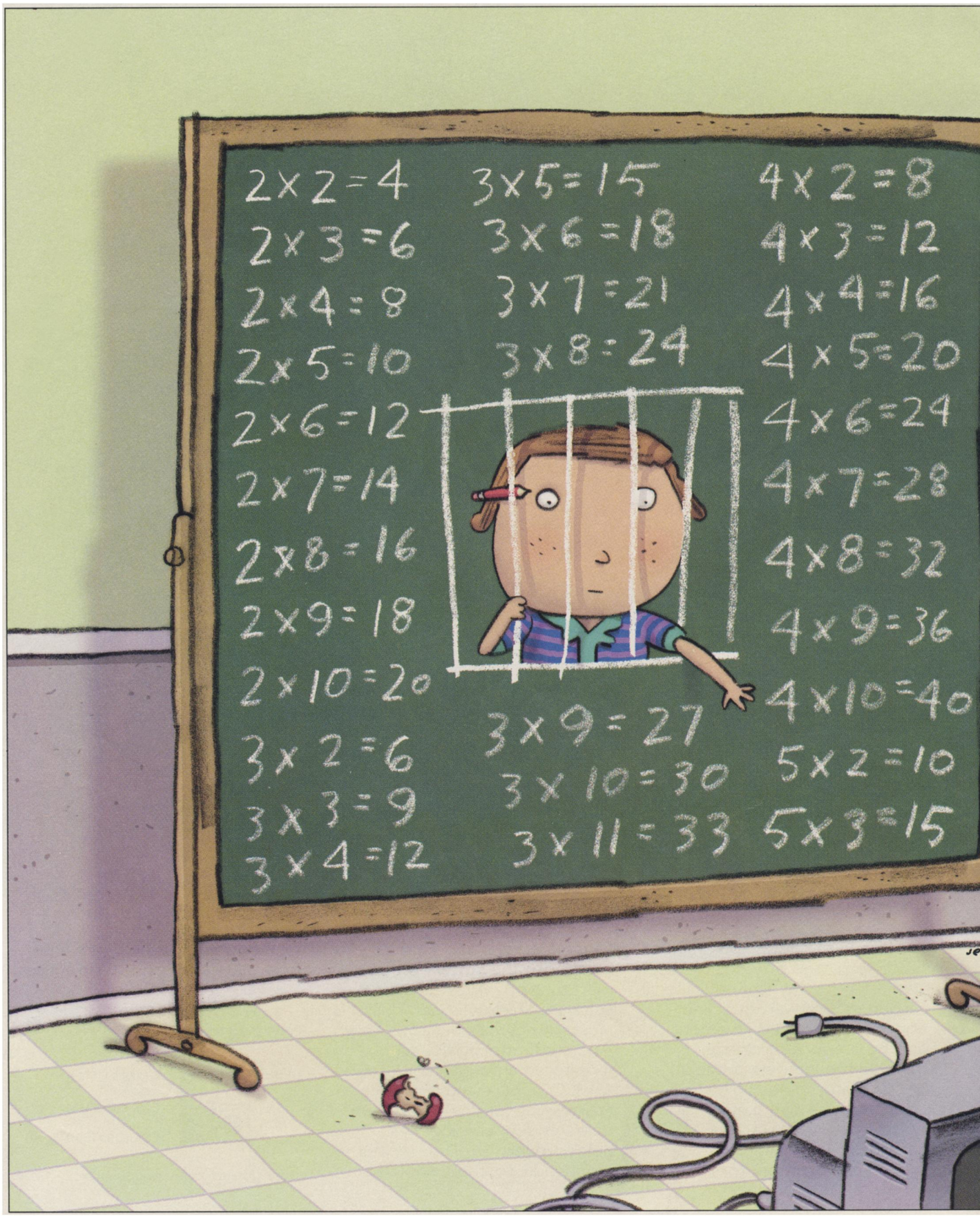
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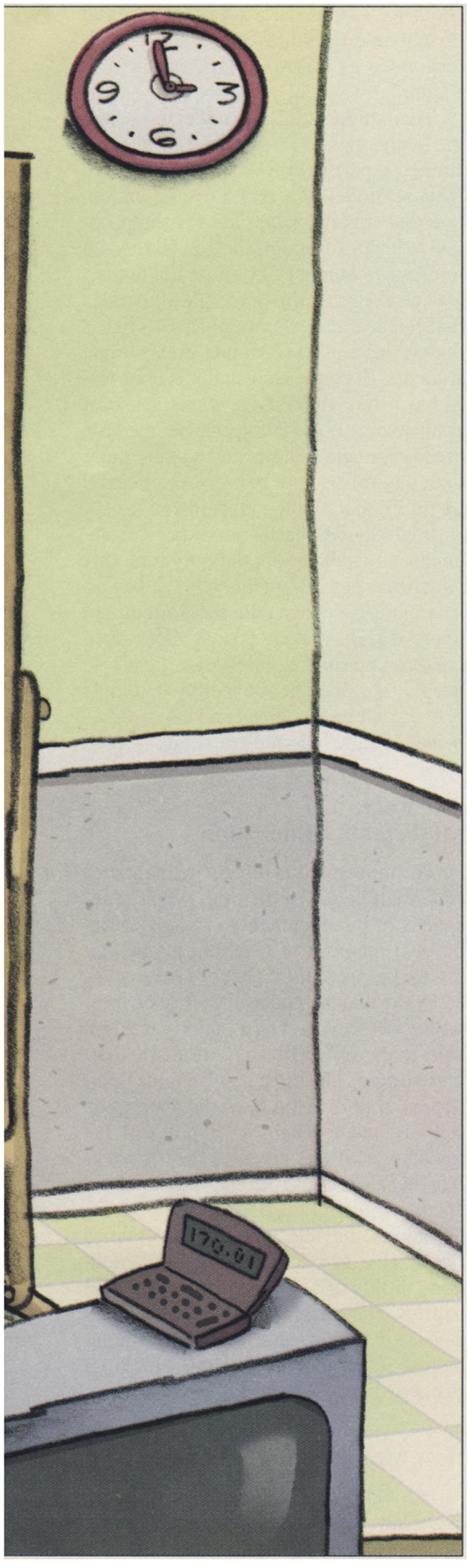

The Mathematical Miseducation of America's Youth

Ignoring Research and Scientific Study in Education

BY MICHAEL T. BATTISTA

To perform a reasonable analysis of the quality of mathematics teaching requires an understanding not only of the essence of mathematics but also of current research about how students learn mathematical ideas, Mr. Battista points out. Without extensive knowledge of both, judgments made about what mathematics should be taught to schoolchildren and how it should be taught are necessarily naive and almost always wrong.

RECENT NEWSPAPER and newsmagazine articles, public debates at local school board meetings, and even the California State Board of Education[1] have aimed a great deal of criticism at the current "reform movement" in mathematics education. Exploiting the growing "talk show/tabloid" mentality of Americans, opponents of reform support their arguments with hearsay, misinformation, sensationalism, polarization, and conflict as they attempt to seize control of school mathematics programs and return them to traditional teaching — that is, to the "basics." As they cite isolated examples of alleged failures of mathematics reform, they ignore the countless failures of traditional curricula. Their arguments lack understanding both of the essence of mathematics and of scientific research on how students learn mathematics.

Unfortunately, flawed as these arguments are, they nonetheless persuade citizens, legislators, and educational decision makers to adopt policies that are inconsistent with relevant professional, scholarly, and scientific recommendations about mathematics teaching. Consequently, they threaten the quality of the mathematics education received not only by the general citizenry but also by future mathematicians, scientists, and engineers. Thus they endanger the entire scientific/technical infrastructure of our country. In this article, I analyze the issues that are relevant to the reform of mathematics education from the perspective of the scholarly analysis that undergirds the reform movement and the current scientific research on mathematics learning.

Traditional Teaching

How would you react if your doctor treated you or your children with methods that were 10 to 15 years out-of-date, ignored current scientific findings about diseases and medical treatments, and con-

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Illustration by Jem Sullivan

IRONICALLY, THE ONLY TIME THAT AMERICANS PAY ANY ATTENTION TO MATHEMATICS TEACHING IS WHEN EDUCATORS ATTEMPT TO IMPROVE IT.

tradicted all professional recommendations for practice? It is highly unlikely that you would passively ignore such practice.

Yet that is exactly what happens with traditional mathematics teaching, which is still the norm in our nation's schools. For most students, school mathematics is an endless sequence of memorizing and forgetting facts and procedures that make little sense to them. Though the same topics are taught and retaught year after year, the students do not learn them. Numerous scientific studies have shown that traditional methods of teaching mathematics not only are ineffective but also seriously stunt the growth of students' mathematical reasoning and problem-solving skills.[2] Traditional methods ignore recommendations by professional organizations in mathematics education, and they ignore modern scientific research on how children learn mathematics. Yet traditional teaching continues, taking its toll on the nation and on individuals.

For the nation, the economic costs of the traditional system of mathematical miseducation are staggering. According to the National Research Council, 60% of college mathematics enrollments are in courses ordinarily taught in high school,[3] and the business sector spends as much on remedial mathematics education for employees as is spent on mathematics education in schools, colleges, and universities combined. The mathematical ignorance of our citizenry seriously handicaps our nation in a competitive and increasingly technological global marketplace.

For individuals, the effects of mathematical miseducation are like a long-term hidden illness that gradually incapacitates its victims. The results of testing by the National Assessment of Educational Progress indicate that only about 13% to 16% of 12th-graders are proficient in mathematics.[4] And according to the National Research Council, 75% of Americans stop studying mathematics before they complete career or job prerequisites.[5] Indeed, although virtually all students enter school mathematically healthy and enjoying mathematics as they solve problems in ways that make sense to them, most exit school apprehensive and unsure about doing all but the most trivial mathematical tasks.

Mathematics anxiety is widespread. So rampant is innumeracy that there is little stigma attached to it. Many adults readily confess, "I was never good at math," as if displaying a badge of courage for enduring what for them was a painful and useless experience. In contrast, people do not freely admit that they can't read.

Of course, although most people acknowledge that numerous students have difficulty with mathematics, they take solace in the belief that bright students are doing just fine. This belief, too, is unfounded. Indeed, because really bright students generally learn symbolic algorithms quickly, they appear to be doing fine when their performance is measured by standard mathematics tests. But a closer look reveals that they too are being dramatically affected by the mathematics miseducation of traditional curricula. For instance, a bright eighth-grader who was three weeks from completing a standard course in high school geometry — thus she was two years ahead of schedule for college-prep students — responded as follows on the problem in Figure 1.[6]

This student did not understand that the mathematical formula she applied assumed a particularly structured mathematical model of a real-world situation, one that was inappropriate for the problem at hand. Although she had learned an impressive number of routine mathematical procedures, this example illustrates that her learning was only superficial, a finding that is all too common among bright students. Because such students obviously have the capability to make sense of mathematics if given the chance, the case could be made that these students, more than any others, are being shortchanged by traditional mathematics instruction.

Ironically, despite this pandemic of mathematics miseducation, the only time that Americans pay any attention to mathematics teaching is when educators attempt to improve it. But misconceptions about mathematics and mathematics learning are so deeply ingrained in our society that most people can't truly comprehend the improvements, so they fear and resist them.

The Reform of Mathematics Education

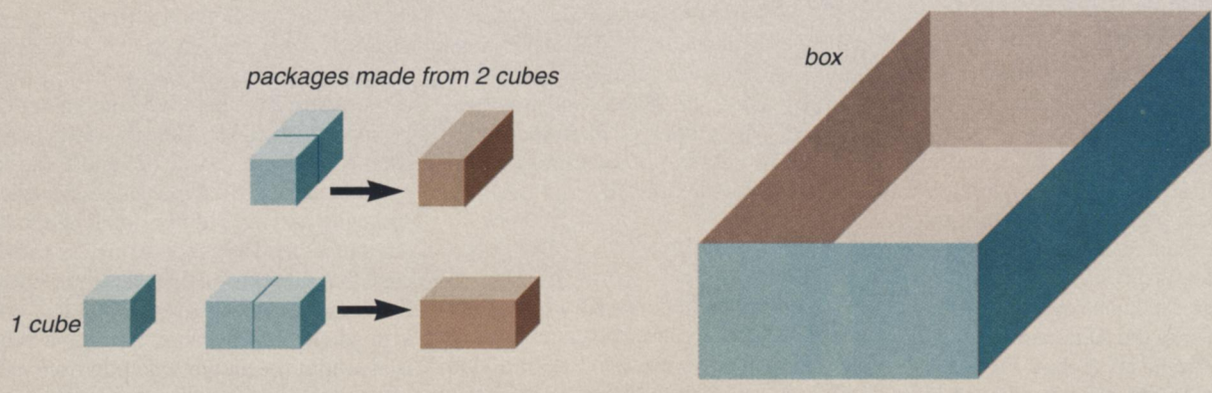
The movement to reform mathematics education began in the mid-1980s in response to the documented failure of traditional methods of teaching mathematics, to the curriculum changes necessitated by the widespread availability of computing devices, and to a major paradigm shift in the scientific study of mathematics learning. The most conspicuous component of reform has been the attempt by schools and teachers to implement the recommendations given in the *Curriculum and Evaluation Standards for School Mathematics*, published by the National Council of Teachers of Mathematics (NCTM) in 1989.[7] Reform recommendations in this and related documents deal with how mathematics is taught, what mathematics is taught, and, at a more fundamental level, the very nature of school mathematics.

How Mathematics Is Taught

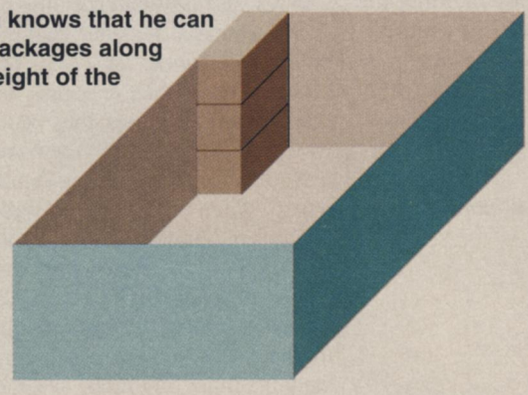
In traditional mathematics instruction, every day is the same: the teacher shows students several examples of how to solve

FIGURE 1.
Student Response to Package Question

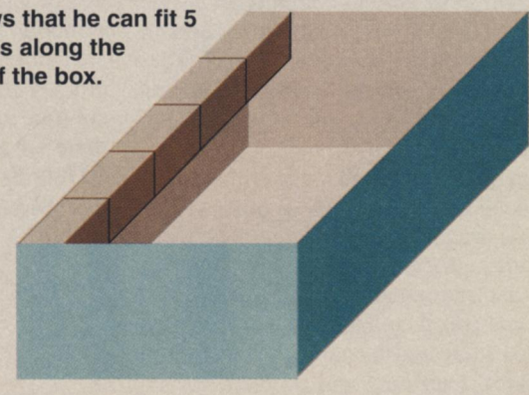
Collin has some packages that each contain two identical cubes. He wants to know how many of these *packages* it takes to completely fill the rectangular box below.



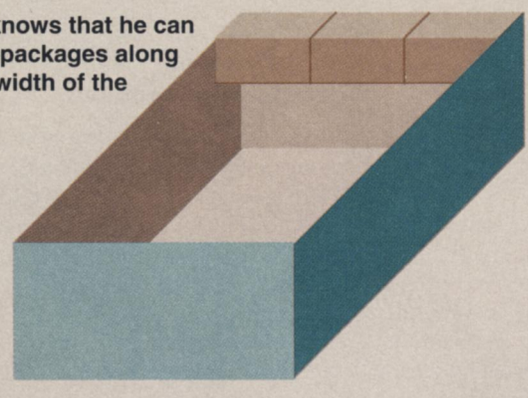
Collin knows that he can fit 3 packages along the height of the box.



He knows that he can fit 5 packages along the length of the box.



He knows that he can fit 3 packages along the width of the box.



Student: *It's 45 packages. And the way I found it is I multiplied how many packages could fit in the height by the number in the width, which is 3 times 3 equals 9. Then I took that and multiplied it by the length, which is 5, and came up with 9 times 5, which is 45.*

Obs: *How do you know that is the right answer?*

Student: *Because the equation of the volume of a box is length times width times height.*

Obs: *Do you know why that equation works?*

Student: *Because you are covering all three dimensions, I think. I'm not really sure. I just know the equation.*

a certain type of problem and then has them practice this method in class and in homework. The National Research Council has dubbed the "learning" produced by such instruction as "mindless mimicry mathematics."[8] Instead of understanding what they are doing, students parrot what they have seen and heard.

In the classroom environment envisioned by NCTM, teachers provide students with numerous opportunities to solve complex and interesting problems; to read, write, and discuss mathematics; and to formulate and test the validity of personally constructed mathematical ideas so that

they can draw their own conclusions. Students use demonstrations, drawings, and real-world objects — as well as formal mathematical and logical arguments — to convince themselves and their peers of the validity of their solutions.

What Mathematics Is Taught

In traditional mathematics instruction, the mathematics covered is almost identical to what most adults were taught when they were children. Students spend most of their time attempting to learn traditional computational procedures — that is, things that can be done on a calculator. Furthermore, the focus on computation is so myopic that few students develop any understanding of why the computations work or when they should be applied. For instance, traditionally taught students who are lucky enough to be able to compute an answer to $2\frac{1}{2} \div \frac{1}{4}$ can rarely explain or demonstrate why their answer is correct. Their explanations usually amount to saying, "My teacher said we were supposed to invert and multiply."

In the mathematics curricula recommended by NCTM and all other professional organizations that deal with mathematics education, the exclusive emphasis that traditional teaching places on paper-and-pencil computation has been moderated. Increased attention is given to mathematical reasoning and problem solving as well as to previously neglected topics, such as statistics and the use of computational devices in mathematical analysis. These curricula focus on the basic skills of *today*, not those of 40 years ago. Problem solving, reasoning, justifying ideas, making sense of complex situations, and learning new ideas independently — not paper-and-pencil computation — are now critical skills for all Americans. In the Information Age and the web era, obtaining the facts is not the problem; analyzing and making sense of them is.

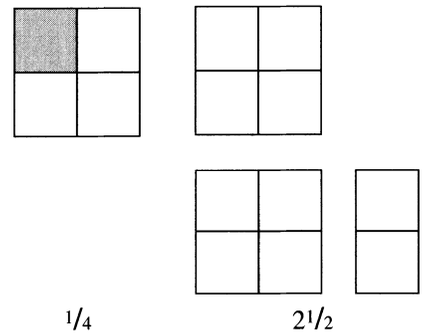
The Nature of School Mathematics

Mathematics is first and foremost a form of reasoning. In the context of reasoning analytically about particular types of quantitative and spatial phenomena, mathematics consists of thinking in a logical manner, formulating and testing conjectures, making sense of things, and forming and justifying judgments, inferences, and conclusions. We do mathematics when we recognize and describe patterns; construct physical and/or conceptual models of phenomena; create symbol systems to help us represent, manipulate, and reflect on ideas; and invent procedures to solve problems.

To illustrate, consider the problem, "What is $2\frac{1}{2}$ divided by $\frac{1}{4}$?" Students taught traditionally are trained to solve such problems by using the "invert and multiply" method, which most of them memorize, quickly forget, and almost never understand. Thus students will write:

$$2\tfrac{1}{2} \div \tfrac{1}{4} = \tfrac{5}{2} \times \tfrac{4}{1}$$

In contrast, students who have made sense of fractions and who understand the operation of division don't need a symbolic algorithm to compute an answer to this problem. Because they interpret the symbolic statement in terms of appropriate mental models of quantities, they are quickly able to reason that, because there are four fourths in each unit and because there are two fourths in a half, there are 10 fourths in $2\frac{1}{2}$. Younger students might need to draw a picture to support such reasoning.



Students who truly make sense of this situation are not manipulating symbols, oblivious to what they represent. Instead, they are purposefully and meaningfully reasoning about quantities. They are not blindly following rules invented by others. Instead, they are making personal sense of the ideas. These students have developed powerful conceptual structures and patterns of reasoning that enable them to apply their mathematical knowledge and understanding to numerous real-world situations, giving them intellectual autonomy in their mathematical reasoning.

Obviously, not all problems can be easily solved using such intuitively appealing strategies. Students must also develop understanding of and facility with symbolic manipulations and even an appreciation for the workings of axiomatic systems that describe how to deal formally with mathematical symbols. Thus it is not enough to involve students only in sense making, reasoning, and the creation of new mathematical knowledge. Sound curricula must include clear long-range goals for ensuring that students become *fluent* in employing those abstract concepts and mathematical perspectives that our culture has found most useful. Students should be able to apply, readily and correctly, important mathematical strategies and lines of reasoning in numerous situations. They should possess knowledge that supports mathematical reasoning. For instance, students should know the "basic number facts" because such knowledge is essential for mental computation, estimation, performance of computational procedures, and problem solving.

Nonetheless, students' learning of symbolic manipulations must never become disconnected from their reasoning about quantities. For when it does, they become overwhelmed with trying to memorize countless rules for manipulating symbols. Even worse, when students lose sight of what symbol manipulations imply about real-world quantities, doing mathematics becomes an academic ritual that has no real-world usefulness. Indeed, to be able to use mathematics to make sense of the world, students must first make sense of mathematics.

The Science of Learning Mathematics

The redefinition of school mathematics curricula and instruction has occurred at the same time as — and, indeed, has been influenced by — the abandonment of the outdated and simplistic behaviorist learning theory that has dictated the course of mathematics teaching for more than 40 years. Mathematics education is struggling mightily to emerge from an era in which the prevailing views of mathematics and learning have been mutually reinforcing: school mathematics has been seen as a set of computational skills; mathematics learning has been seen as progressing through carefully scripted schedules of acquiring those skills. According to the traditional view, students acquire mathematical skills by imitating demonstrations by the teacher and the textbook. They acquire mathematical concepts by "absorbing" teacher

MOST EDUCATORS AND ALMOST ALL NONEDUCATORS HAVE NO SUBSTANTIVE UNDERSTANDING OF THE RESEARCH-BASED CONSTRUCTIVIST THEORY.

and textbook communications.

In contrast, all current major scientific theories describing students' mathematics learning agree that mathematical ideas must be personally constructed by students as they try to make sense of situations (including, of course, communications from others and from textbooks). Support for the basic tenets of this "constructivist" view comes from the noted psychologist Jean Piaget and, more recently, from scientists attempting to connect brain function to psychology. For instance, Nobel laureate Francis Crick has stated, "Seeing is a constructive process, meaning that the brain does not passively record the incoming visual information. It actively seeks to interpret it."[9] Similarly, psychologist Robert Ornstein asserts, "Our experiences, percepts, memories are not of the world directly but are our own creation, a dream of the world, one that evolved to produce just enough information for us to adapt to local circumstances."[10]

More than two decades of scientific research in mathematics education have refined the constructivist view of mathematics learning to provide detailed explanations of how students construct increasingly sophisticated ideas about particular mathematical topics, of what students' mathematical experiences are like, of what mental operations give rise to those experiences, and of the sociocultural factors that affect students' construction of mathematical meaning. To distinguish this theory, which is based on empirical research, from the broad philosophical constructivist stance taken by educators specializing in other disciplines, I will refer to it as "scientific constructivism."

Unfortunately, most educators (including many teachers, educational administrators, and professors of education) and almost all noneducators (including mathematicians, scientists, and writers for the popular press) have no substantive understanding of the research-based constructivist theory that I have alluded to above. Many of them conceive of constructivism as a pedagogical stance that entails a type of nonrigorous, intellectual anarchy that lets students pursue whatever interests them and invent and use any mathematical methods they wish, whether these methods are correct or not. Others take constructivism to be synonymous with "discovery learning" from the era of "new math," and still others even see it as a way of teaching that focuses on using manipulatives or cooperative learning.

None of these conceptions is correct. Scientific constructivism is a well-developed scientific theory that has proved invaluable in understanding empirical research on students' learning of mathematics. To illustrate some of the depth of scientific constructivism, I briefly discuss its description of two fundamental learning mechanisms and offer an example of the type of insight that can result from constructivist research.

Abstraction, Reflection, And Learning

In scientific constructivist accounts of learning, abstraction is the fundamental mental mechanism by which new mathematical knowledge is generated. Abstraction is the process by which the mind selects, coordinates, combines, and registers in memory a collection of mental items or acts that appear in the attentional field. There are different degrees of abstraction, ranging from isolating an item in the experiential flow and grasping it as a unit to disembedding it from its original perceptual context so that it can be freely operated on in the imagination, including projecting it into other perceptual material and using it in novel situations. Although the process of abstraction has been discussed for centuries, current scientific constructivist research is elaborating its exact role in mathematics learning.[11] Meanwhile, neuroscience is beginning to contemplate the workings of abstraction in the brain.[12] Accounts from both camps make it clear that abstraction is the critical mechanism that enables the mind to construct the mental entities that individuals use to reason about their "mathematical realities."

Understanding mathematics, however, requires more than abstraction. It requires *reflection*, which is the conscious process of mentally replaying experiences, actions, or mental processes and considering their results or how they are composed. As these acts of reflection are themselves abstracted, they can become the content — what is acted upon — in future acts of reflection and abstraction.

What emerges from this theory is a picture of meaningful mathematics learning coming about as individuals recursively cycle through phases of action (physical and mental), reflection, and abstraction in a way that enables them to integrate related abstractions into ever more sophisticated mental models of phenomena. In fact, students' ability to understand and effectively use the formal mathematical systems of our culture to make sense of their quantitative and spatial surroundings depends on their construction of elaborated sequences of mental models. Initial models in these sequences enable students working with real-world objects to reason about their physical manipulations. Later models permit them to reason with mental images of real-world objects. Finally, symbolic models enable them to reason by meaningfully manipulating mathematical symbols that represent real-world situations.

Without this recursively developed sequence of mental models, students' learning about mathematical symbol systems is strictly syntactic, and their use of symbolic procedures is totally disconnected from real-world situations. Research has

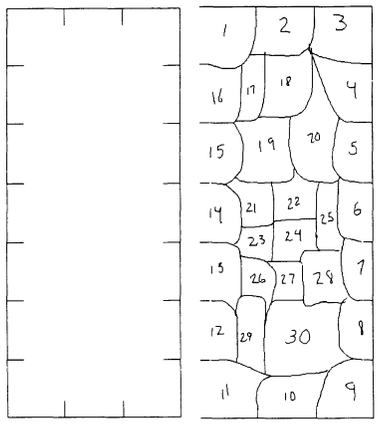
shown repeatedly that rote learning of syntactic rules for manipulating symbols is exactly what results for most students in traditional mathematics curricula.

Attending to Students' Mathematical Constructions

Although the description I have just given illustrates some of the conceptual depth of a scientific constructivist view of mathematics learning, there is much more to the constructivist research program than its general description of learning. In fact, a careful reading of constructivist literature reveals that the power and usefulness of the theory resides not in its general formulation but rather in the particulars and refinements of its application. Contemporary constructivist researchers in mathematics education have gone well beyond the general theory to develop specific models of students' ways of operating as they construct increasingly sophisticated mathematical knowledge in particular mathematical situations. It is this elaboration and particularization of the general theory that makes this research truly relevant to instructional issues.

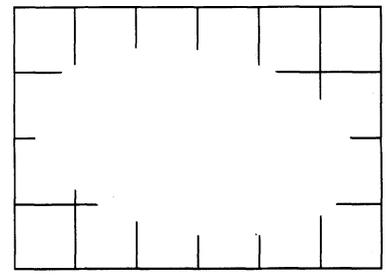
An example from elementary school mathematics illustrates the kinds of insights that can be gained by carefully examining students' construction of particular mathematical ideas.[13] CS, a second-grader, was shown a one-inch plastic square and the 3-inch by 7-inch rectangle illustrated in Figure 2. She was also shown that the plastic square was the same size as one of the squares on the rectangle.

FIGURE 2.

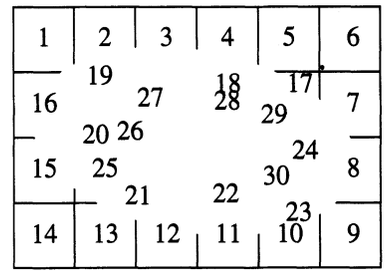


CS was then asked to predict how many of the plastic squares it would take to completely cover the rectangle. She drew squares where she thought they would go and counted 30, as shown in Figure 2.

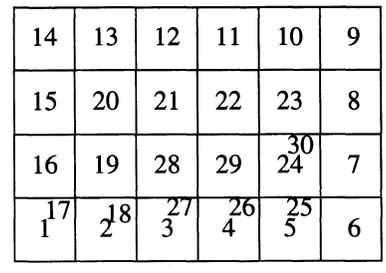
On a similar problem, CS was asked to predict how many squares would cover the rectangle shown in this figure.



This time, however, she was asked to make a prediction without drawing. CS pointed and counted as shown in the following figure, predicting 30.



When checking her answer with plastic squares, she pointed to and counted squares as shown in the next figure, getting 30. But she got confused, so she counted again, getting 24, then 27.



Clearly, CS was not imagining the row-by-column organization that most adults "see" in these rectangular arrays of squares. Although, as educated adults, we easily see how rows and columns of squares will cover these rectangles, CS had not yet mentally constructed this organization. For her, this row-by-column organization wasn't there. It simply didn't exist.

And CS's thinking is not unusual. Research shows that only 19% of second-graders, 31% of third-graders, 54% of fourth-graders, and 78% of fifth-graders make correct predictions about how many unit squares will cover a rectangle. These are sobering findings, given that for students in these grades, traditional instruction uses rectangular arrays as a model to give meaning to multiplication, *assuming* that students see such arrays as sets of equivalent columns and rows. To construct a proper row-by-column structuring of such arrays, these students must spatially coordinate the elements in the orthogonal dimensions of rows and columns, something that is quite difficult for many of them.[14]

Proper Mathematics Instruction

To be consistent with the scientifically based constructivist theory that I have described, mathematics teaching must use detailed scientific research on how students construct particular mathematical ideas to guide and nurture their personal construction of mathematical ideas. Because traditional instruction ignores students' personal construction of mathematical meaning, the development of their mathematical thought is not properly nurtured, resulting in stunted growth.

One distressing illustration of this phenomenon can be seen when we examine the mathematical conceptions of the growing number of college students who have difficulty with basic university mathematics courses. When all is said and done, we find that these students have forgotten most of the formal mathematics they "learned" beyond elementary school and have reverted to intuitive conceptions that they developed before reaching adolescence. Because the formal mathematics they learned in school was disconnected from these intuitive notions, not only have the intuitive notions gone undeveloped, but the formal mathematics has made little sense. Thus it was seen as not having much use and was quickly forgotten. The career aspirations of these students — many of whom are quite capable in other academic areas — are dashed or seriously jeopardized.

To develop powerful mathematical thinking in students, instruction must focus on, guide, and support their personal construction of ideas. Such instruction encourages students to invent, test, and refine their own ideas rather than to blindly follow procedures given to them by others. For example, returning to the fraction example described above, if students are going to progress to a meaningful understanding of the symbolic manipulation of fractions, that understanding must come

from students' reflections on their own work with physical fractional quantities. Given appropriate experiences in mentally manipulating these quantities, students can, with proper guidance, derive strictly symbolic methods for dividing fractions. They might invent the "invert and multiply" method, or they might come up with a different symbolic procedure. (For instance, some students get a common denominator and then divide the numerators.) Because students derive these symbolic procedures through personally meaningful manipulation of quantities, their knowledge of the procedures becomes semantically rich in its connection to their reasoning about quantities. It is no longer inert and strictly syntactical. Research clearly shows that such "construction-focused" mathematics instruction produces more powerful mathematical thinkers.[15]

Genuine Issues in Improving Mathematics Learning

Because opponents of reform have sensationalized the mathematics education debate and turned it into a naive "basics and tradition are good — reform is bad" dichotomy, their attacks have obscured the genuine issues that require careful analysis. I now briefly outline several of these issues.

Lack of knowledge. The major impediment to improving students' mathematics learning is adults' lack of knowledge — both of mathematics and of research on how students learn mathematics. Because mathematics has been taught so poorly for so long, few adults have a genuine understanding of mathematics or of the mathematical enterprise. Most adults, who have been mathematically miseducated themselves, believe that mathematics is the performance of set procedures invented by others. They have learned — and expect others to learn — mathematics as a set of rigid rules invented by others. They simply do not understand mathematics well enough to appreciate when it has been learned well. The situation is worse when it comes to scientific knowledge about students' mathematics learning. As a consequence, it is extremely difficult for school districts to implement authentic reform because teachers and administrators not only must educate students but also must reeducate parents to understand and support reform.

Disregard of science. One of the major reasons that American educational practice in general and mathematics education in particular have made so little progress is that they have failed to adhere to scientific methodology. Too often the educational programs and methods used in schools are formulated — by practitioners, administrators, laypeople, politicians, and professors of education — with a total disregard for scientific research. Because educational practice is not subject to the critical scrutiny of scientific analysis and review, educators continually "reinvent the wheel." They follow one bandwagon after another. In fact, Kenneth Wilson and Bennett Daviss liken the current state of educational curriculum development in the U.S. to that of aeronautics before the Wright brothers.

> A century ago, people making airplanes were usually solitary, self-taught visionaries or eccentrics following their own theories or hunches. They lacked a good deal of information about aerodynamics. . . . They continued to work separately, often unknowingly crossing and recrossing each other's tracks, unable to take advantage of or build on each other's successes.[16]

As a consequence of education's disregard for scientific practice and the resulting failure to improve student learning, the general public has little faith in the ability of professional educators to steer the educational enterprise wisely. Individuals in all walks of life value their personal opinions about education as much as or more than those of professional educators. It even happens within the field of education itself. For instance, professors of education whose area of expertise has nothing to do with mathematics often feel free to make grand pronouncements about how mathematics should be taught. Furthermore, because of the lack of confidence in professional educators, control of educational programs is often taken out of their hands. As in California and in many local school districts across the nation, the effort to control school curricula becomes a heated political battlefield where scientific reasoning plays no role.

To steer the educational enterprise away from its current state of chaos, educational practice must be based on established scientific research about how students learn. Knowledge obtained by such research is more reliable than the commonsensical ideas and folk wisdom most people use to make judgments about teaching. Knowledge obtained scientifically is constructed according to rigorous standards of reasoning and verification upheld by scientific communities of scholars who constantly review, test, critique, and build on each others' work. Because it is developed so carefully, scientific knowledge is held in high esteem by most educated members of our culture. Thus relying on scientific knowledge can serve as a focal point for consensus building in rational discussions of educational practice.

Who the scientific researchers are. If we wish to heed scientific research in mathematics education, to whom should we turn for findings from this research? As always, we should consult specialists; we should look to scientific researchers whose specialty is research in mathematics education. As obvious as this seems, it is usually ignored by opponents of mathematics reform. Because they don't agree with the findings of the specialists, they seek out researchers in other areas to buttress their case. For instance, there are educational and cognitive psychologists who occasionally conduct research on the learning of mathematics. Unfortunately, they usually apply general, essentially behaviorist theories that ignore both the methods and the results of modern mathematics education research. Research conducted by these nonspecialists is so out of step with state-of-the-art mathematics education research that relying on its results sets back one's conception of mathematics learning and teaching at least two decades. Taking a scientific approach to designing appropriate mathematics instruction requires one to examine state-of-the-art research conducted by specialists, not out-of-date research performed by interlopers.

The myth of coverage. One of the major consequences of the blatant disregard of modern scientific research on mathematics learning is the almost universal belief in what I call the "myth of coverage." According to this myth, "If mathematics is 'covered,' students will learn it." The myth is not restricted to mathematics alone, of course. But this myth is so deeply embedded in traditional mathematics instruction that at each grade level teachers feel tremendous pressure to cover huge amounts of material at breakneck speeds. Furthermore, belief in the myth causes teachers to criticize reform curricula as inefficient because students in such curricula study far fewer topics at each grade level.

Basing his conclusions on scientific

research, Alan Bell of the Shell Centre for Mathematical Education at the University of Nottingham counters this myth-based reasoning as follows:

> It may be felt that there is no time for a method which involves intensive discussion of particular points. But on the evidence presented . . . we have to ask whether we can afford to waste pupils' time on [traditional] methods which have such little long-term effect when . . . we could be doing so much better.[17]

That is, because students in traditional curricula learn ideas and procedures by rote (if at all) rather than meaningfully, they quickly forget them, so the ideas must be retaught year after year. In sense-making curricula, on the other hand, because students retain learned ideas for long periods of time, and because a natural part of sense making is to interrelate ideas, students accumulate an ever-increasing store of well-integrated knowledge. Indeed, consistent with Bell's claim, the TIMSS data suggest that Japanese teachers, whose students significantly outperform U.S. students in mathematics, spend much more time than U.S. teachers having students delve deeply into mathematical ideas.[18]

Putting scientific research aside, most teachers have plenty of personal experience that contradicts the myth of coverage. How many times, several weeks after teaching a mathematical topic, do teachers return to the topic and find their students acting as if they had never seen it before? How many times do teachers at one grade level find students totally ignorant of mathematical topics "covered" during the previous year — even claiming that they never saw the topics before? As a deep-seated dogma of traditional mathematics instruction, belief in the myth of coverage seems impervious to reasoned analysis.

Testing. Most school districts rely heavily on standardized tests and state "proficiency" tests as bottom-line measures of their students' progress in learning. This practice has several untoward consequences. First, if the tests measure traditional outcomes — and many still do — their use maintains the inertia of traditional instruction and seriously impedes the adoption of reform. Second, such tests are rarely consistent with scientific research on what mathematical understandings should be expected of students at various grade levels. Consequently, teachers, guided by the myth of coverage and pressured by administrators and parents to ensure that students pass such high-stakes tests, often demand that students use abstract mathematical procedures that they can't understand in any meaningful way. These students haven't had enough opportunities to construct through experience the appropriate mental models to serve as the foundation for such abstract learning. Students are thus forced either to "drop out" of the study of mathematics or to resort to mindless mimicry.

Finally, poor understanding of the process of testing creates the "teach to the test" phenomenon that is observed in so many school systems. Because of state-mandated proficiency tests, instead of teaching mathematical concepts and reasoning, most school programs teach students how to solve by rote the specific types of problems that appear on these tests. In fact, one mathematics educator tells the story of a school district's mathematics supervisor who noticed that, on the state proficiency test, area problems had shaded figures and perimeter problems did not. Subsequently, he told teachers to teach students to multiply the dimensions when they saw shaded rectangles and to add them for unshaded rectangles.

While teaching to the test is rarely so blatantly dishonest, it always reduces the curriculum to mimicry mathematics. Moreover, such teaching invalidates large portions of the tests. Indeed, to assess genuine *understanding* of a concept, test items must assess whether students can apply their knowledge in novel situations. If teachers teach students rote procedures for doing these novel items, then the items no longer test understanding, but only mere memorization.

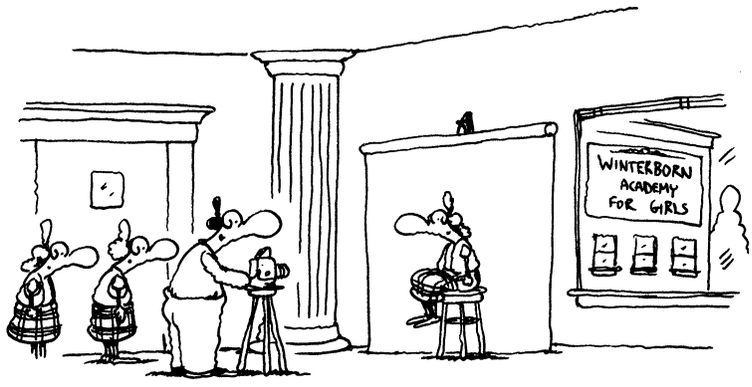
Dilutions and distortions. One criticism often leveled at the mathematics education reform movement is that the ideas it proposes are untested. This is an important point that must be dealt with carefully, for the real answer is not as straightforward as either some opponents or some proponents of reform might have us believe. First, we must examine the scientific basis for reform. Extensive studies have shown not only that traditional teaching is ineffective, but also what is wrong with it. Still other studies have shown that instruction that is consistent with the basic principles of scientific constructivism is more effective than traditional teaching. In fact, through a broad spectrum of studies, the constructivist view of learning and teaching that I described above has been scientifically established; "constructivism" has become the dominant theoretical position among mathematics education researchers.[19] Although, as with all scientific theories, this theory requires further elaboration, testing, and refinement, it is far and away the best analysis we have ever had of students' mathematics learning. Consequently, mathematics teaching that implements scientific constructivism with high fidelity is *not* based on untested theory.

However, the critical question is, To what extent is scientific constructivism being implemented in current mathematics curricula? At this time, I know of no commercially available mathematics curricula that are systematically and completely based on scientific constructivism. Even NCTM's *Curriculum and Evaluation Standards for School Mathematics* is not completely consistent with scientific constructivism, embracing its general tenets but ignoring many of its particulars. (This should not be surprising, since the standards were developed before many of the details of the theory had been worked out.)

Nevertheless, the curricula that come closest to implementing scientific constructivism are those that were developed, with support from the National Science Foundation, specifically to implement the NCTM *Standards*. And because these curricula were tested in a wide variety of classrooms and to the extent that the *Standards* are based on the basic tenets of constructivism, they surely are not "untested."

However, this is the first point at which the dilution of scientific theory may have occurred in practice. When it comes to instructional units to teach particular mathematical topics, even the most accomplished curriculum developers do not pay adequate attention to research on how students learn those topics. Worse yet, even with reform-consistent curricula, teachers with incorrect conceptions of and beliefs about mathematics or about how mathematics is learned can completely distort the original ideas of the curricula's creators, turning dilution into outright distortion.[20]

Next we consider curricula produced by publishing companies. Because such companies are profit-making organizations, they publish what will sell, regardless of scientific research on students' mathematics learning. For example, if the department of education in California demands textbooks that focus on huge amounts of



"Say 'Brie.'"

drill and practice, then, because of that state's large population, all the major textbook companies will produce such texts. Thus, although almost all commercially available mathematics textbooks claim to be consistent with the NCTM *Standards,* most of these textbooks consist of traditional curricula with enough superficial changes tacked on so that publishing companies can market them as "new" and consistent with reform. For the most part, textbook companies have produced mathematical curricula that are mere caricatures of genuine reform curricula. At this point we have outright distortion of reform principles.

Thus, while many school districts claim to be implementing curricula based on mathematics reform, their implementations often distort the tenets of reform so greatly and are so far removed from the scientific research on mathematics learning that the efforts cannot truly be considered reform mathematics at all. As a consequence, great care must be taken in evaluating school districts' "implementations of reform." Just because a particular implementation fails does not mean that one can reasonably conclude that the theory and the research are wrong. One can conclude only that mechanisms for putting theory into practice — teacher preparation, inservice training, textbook creation, and teaching — may be flawed.

In addition, we should not expect even authentic reform efforts to be perfect. Although the curricula of reform have been tested in actual classrooms, because funding agencies did not support the projects that developed them long enough for long-term assessment and revision and because the curricula were first attempts at substantive reform, their extended use is bound to reveal needed alterations and refinements. However, instead of reacting to perceived failures by "throwing the baby out with the bath water," we should work together to find better ways to implement sound scientific theory. We do not need to go back to traditional methods that research and experience have shown do not work.

To perform a reasonable analysis of the quality of mathematics teaching requires an understanding not only of the essence of mathematics but also of current research about how students learn mathematical ideas. Without extensive knowledge of both, judgments made about what mathematics should be taught to schoolchildren and how it should be taught are necessarily naive and almost always wrong. Just as medical treatment must be based on what current research tells us about disease and healing, mathematics teaching must be based on what current scientific research tells us about how students learn mathematics. We must take mathematics curriculum decisions out of the political arena and place them in the hands of professional mathematics educators.

However, giving educators more power to control mathematics curricula requires that they act much more responsibly than they have in the past. We must demand that educators at all levels make their practice consistent with scientific findings and principles. We can no longer afford to permit the educational enterprise to squander its precious human capital.

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